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BREAKAWAY FLOW AROUND GRIDS OF NONCIRCULAR TUBES

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On the basis of the discrete-vortex method, breakaway flow around a single-row assembly (grid) of tubes of square, rectangular, and triangular cross section is investigated.

1. Loss of pressure (resistance) and heat transfer in the transverse flow around tube assemblies are determined primarily by the character of the fluid flow close to the tube surfaces, which, in turn, depends on the geometric parameters of the assembly (the shape of the tube cross section, their distance apart, etc.) and on the conditions of flow around the assembly (Reynolds number Re). In the range of Reynolds numbers characteristic in practice, flow around the tubes is always of breakaway type and is accompanied by the formation of a developed accompanying wake behind the tubes [1].

The difficulty in solving the complete Navier–Stokes or Reynolds equations for describing breakaway flow conditions at bodies with limitingly high Re, when the influence of molecular viscosity on the flow is slight, has led to the development of calculation methods based on the model of an ideal medium. An example of the realization of this approach is the currently widespread discrete-vortex method [2]. The agreement between the calculation results obtained by this method and experimental data provides the basis for the assumption that this approach is justified in considering completely developed turbulent flow, when the flow-breakaway point at the surface of the body is known in advance.

In the present work, results obtained by the discrete-vortex method are given for the resistance of a single-row assembly (grid) of tubes of square, rectangular, and triangular cross section. Grids consisting of plates are also considered. It is supposed that flow breakaway at these bodies occurs for tubes at points of discontinuity of the cross section and for plates at their sharp edges. Note that the calculation results for the flow obtained by the discrete-vortex method are the initial data for calculating the boundary layer at tube surfaces and the heat transfer between the tubes and the flow.

2. The basic assumptions of the discrete-vortex method for calculating various breakaway flows were outlined in [2]; they reduce to the following. The medium is ideal and incompressible. Breakaway is modeled using vortex surfaces that are convergent in the flow. The character of the limiting flow in the general case is established by studying the whole process of flow formation over time.

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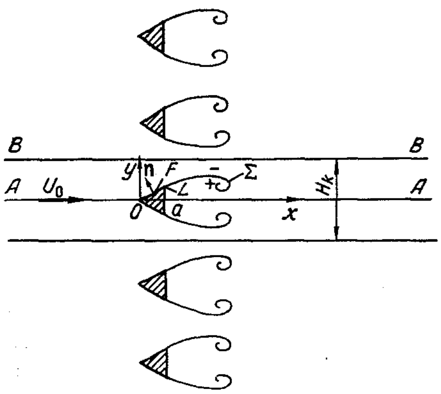

Fig. 1

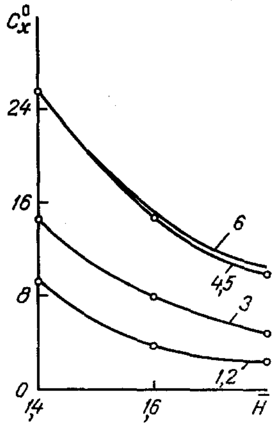

Fig. 2

Fig. 1. Nonsteady breakaway flow around an infinite grid of tubes.

Fig. 2. Dependence of the drag coefficient C_x° on the spacing $\bar{H}$ for grids consisting of rectangular tubes (1), square tubes (2), triangular tubes with their vertices turned toward the incoming flow (3), and triangular tubes with their bases turned toward the incoming flow (4), and theoretical (5) and experimental [4] (6) results for grids made from plates.

The mathematical formulation of the given problem is outlined for the example of nonsteady breakaway flow at an infinite grid of cylindrical tubes of triangular cross section (Fig. 1). One significant assumption adopted in formulating both this and other problems may be noted. The solution of the problem is sought in the class of symmetric functions, i.e., it is assumed that symmetry of the bodies in the grid and of boundary and initial conditions ensures symmetry of the flow. Thus, to solve the problem, it is sufficient to consider the flow arising in the region around one of the symmetric halves of the tube cross section and bounded by the symmetry planes AA and BB.

The mathematical formulation of the problem includes a continuity equation in the form of a Laplace equation for the potential of the relative flow, which is valid everywhere outside the cross section F and its wake Σ; the Chaplygin–Zhukovskii hypothesis of finite velocity at point L of cross section F, where the vortex surface Σ converges with the body; the kinematic continuity condition of the normal velocity component and the dynamic continuity condition of the pressure at the vortex surface Σ; the condition that the fluid does not penetrate through the tube surface and the symmetry surfaces AA and BB. Additionally, the condition at infinite distance from the grid and the wake and also specified initial conditions are used in the given problem.

Numerical realization of this nonlinear model of breakaway flow around bodies is based on the replacement of the continuous vortex surfaces forming the breakaway flow by a system of discrete vortices. This reduces the problem of investigating the integrodifferential equations which describe the behavior of the continuous vortex surface to solving a considerably simpler system of ordinary differential equations. The contour of cross section F of the tube and its wake are replaced by a system of discrete vortices. The boundary condition of impenetrability of the tube surfaces is satisfied at a finite number of control points, whose positions are chosen in accordance with the recommendations in [2]. To satisfy the impenetrability conditions for the symmetry planes AA and BB, a vortex in a plane rectilinear channel is used as the basic vortex element. The velocity field and flow potential induced by this vortex element may be obtained by the method of conformal transformations or the method of successive mappings. Thus, for a discrete vortex with intensity Γ at a point with coordinates x_0, y_0 of a channel of width $H_k/2$, the velocity components and flow potential at point x, y are determined from the formulas

$$V_x = \frac{\Gamma}{2H_k}\left[\frac{\sin\frac{2\pi}{H_k}(y-y_0)}{\operatorname{ch}\frac{2\pi}{H_k}(x-x_0)-\cos\frac{2\pi}{H_k}(y-y_0)} - \frac{\sin\frac{2\pi}{H_k}(y+y_0)}{\operatorname{ch}\frac{2\pi}{H_k}(x-x_0)-\cos\frac{2\pi}{H_k}(y+y_0)}\right];$$

$$V_y = -\frac{\Gamma}{2H_k}\left[\frac{\operatorname{sh}\frac{2\pi}{H_k}(x-x_0)}{\operatorname{ch}\frac{2\pi}{H_k}(x-x_0)-\cos\frac{2\pi}{H_k}(y-y_0)} - \frac{\operatorname{sh}\frac{2\pi}{H_k}(x-x_0)}{\operatorname{ch}\frac{2\pi}{H_k}(x-x_0)-\cos\frac{2\pi}{H_k}(y+y_0)}\right];$$

$$\varphi=\frac{\Gamma}{2\pi}\left(\operatorname{arctg}\left(\operatorname{ctg}\frac{\pi}{H_k}(y-y_0)\operatorname{th}\frac{\pi}{H_k}(x-x_0)\right)-\operatorname{arctg}\left(\operatorname{ctg}\frac{\pi}{H_k}(y+y_0)\operatorname{th}\frac{\pi}{H_k}(x-x_0)\right)+C\right),$$

where $C = 0$, $y \geq y_0$; $C = \pi$, $y < y_0$, $x \geq x_0$; $C = -\pi$, $y < y_0$, $x < x_0$.

The nonsteady problem is solved in stepwise fashion, by passing from one calculational moment of time to another, and reduces, at each calculational moment, to the successive solution of two systems of equations, one of which includes linear algebraic equations, while the other includes ordinary linear differential equations.

The system of algebraic equations is well-founded, and therefore the simple and reliable Gauss method is used for its solution. The system of differential equations describing the motion of free vortices is integrated by the Euler method, with a time step equal to the calculational time interval of free-vortex formation Δt. Regarding Δt, it must be noted that in [2] the determination of the calculational time interval in accordance with the discretization scheme for the body in the flow and the velocity of the incoming flow was recommended. Since the velocity of the incoming flow at the grid may differ considerably from the flow velocity in the vicinity of the breakaway point in the given problem, depending on the grid parameters (and it is the latter velocity which determines the process of vortex formation and erosion), the following formula is proposed for the calculational time interval Δt

$$\Delta t = h/V_{0\tau}^{L},$$

where $V_{0\tau}^{L}$ is the tangential component of the mean flow velocity at the point where the Chaplygin—Zhukovskii hypothesis is satisfied.

From the vortex circulation found at each calculational moment, using the Cauchy—Lagrange integral [3], the pressure distribution over the tube surface is calculated, and by integrating the pressure over the surface the drag coefficient of the grid is found.

3. On the basis of the foregoing method, breakaway flow around grids consisting of tubes of various cross sections has been calculated on a computer. Three forms of tube cross section are studied: equilateral triangles, squares, and rectangles with a base a equal to half the height b; grids consisting of plates are also studied. For triangular tubes, two orientations in the flow are considered: bases and vertices opposite to the incoming flow. All the tubes have the same cross-sectional height b, taken as the characteristic linear dimension in the present work. Depending on the form of the body in the flow, different numbers of discrete vortices ($40 \leq N \leq 80$) are employed in the calculations. The formation of a limiting flow occurs with variation in the velocity of the incoming flow according to the law

$$U(\tau)=0,\ \tau\leqslant 0;\ U(\tau)=U_0,\ \tau>0,$$

where $\tau = U_0 t/b$.

Initially ($\tau < \tau^*$), the vortex sheet converging with the tube surface is a continuous surface of spiral form. Over time, as nonsteady flow develops, the vortex sheet loses stability and breaks down. In the wake, agglomerations of finite size are formed. The quantity τ^* characterizing the onset of sheet breakdown depends weakly on the form of the tube cross section and is determined principally by the grid spacing $H = H_k/b$. Decrease in grid spacing leads to more rapid breakdown of the vortex sheet and the establishment of a discrete structure of the vortex wake. For example, for $\bar{H} = 1.4$, $\tau^* \approx 1.0$; for $\bar{H} = 1.1$, $\tau^* \approx 0.1$.

The aerodynamic loads acting on the grid also vary in accordance with the features of wake development. The dependence $C_x(\tau)$ is characterized by limiting values C_x°, which are established after the end of transient conditions of flow formation in the wake immediately behind the grid. For the given configurations of the tube cross section and grid spacing values $\bar{H}$ =

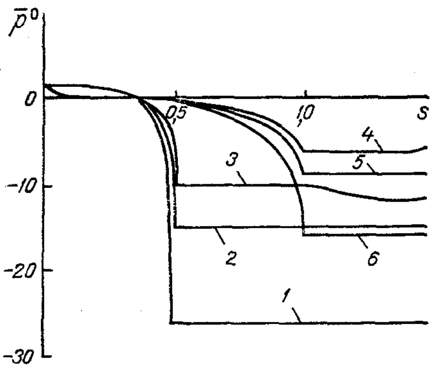

Fig. 3

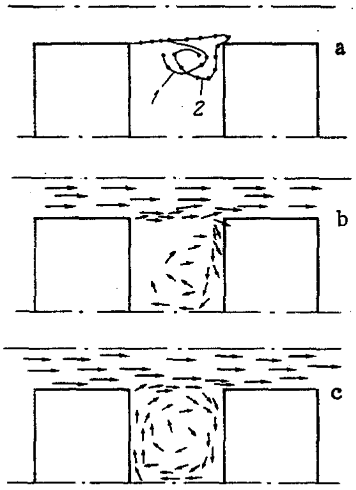

Fig. 4

Fig. 3. Limiting pressure distribution $\bar{p}^0$ over the surface of triangular tubes with their bases (curves 1-3) and their vertices (curves 4-6) turned toward the flow: 1, 6) $\bar{H}$ = 1.4; 2, 5) 1.6; 3, 4) 1.8.

Fig. 4. Development over time of breakaway flow around a bundle of rectangular tubes arranged in a corridor configuration: a) form of vortex sheet at times τ = 0.45 (1) and 0.75 (2); b, c) vector velocity fields at times τ = 1.85 and 2.50, respectively.

1.4, 1.6, 1.8, the time for formation of the wake immediately behind the grid is $\tau \simeq 1.2$. Theoretical values of the drag coefficients of grids made from tubes and plates are shown in Fig. 2, together with experimental data on the resistance of plate grids taken from [4]. As is evident from Fig. 2, the theoretical and experimental results are in good agreement. Decrease in the grid spacing leads to sharp increase in drag coefficient. Grids consisting of triangular tubes with their bases turned toward the incoming flow have the greatest resistance. Note that the resistance of such grids practically coincides with the drag of plate grids.

The influence of the grid parameters on the local aerodynamic characteristics — curves of $\bar{p}^0(s)$ — is shown in Fig. 3 for the example of tubes of triangular cross section. The dimensionless coordinate s varies along the contour of the cross section and is measured from the point x = 0 (leading stagnation point of the flow). As is evident from Fig. 2, the grid spacing and orientation of its elements have a significant influence on the attenuation behind the grid. The pressure along the grid (along the y axis) in the bottom region is practically unchanged here.

Note, in conclusion, that breakaway flow may be studied using the method here considered not only for grids but also for bundles of tubes arranged in a different order, for example, corridor configuration (Fig. 4).

NOTATION

Re, Reynolds number; U_0, flow velocity; x, y, spatial coordinates; t, time; H_k, grid spacing; b, height of tube cross section; τ, dimensionless time; $\bar{H}$, dimensionless grid spacing; N, number of discrete vortices; Δt, calculational time step; h, vortex-scheme step; $\bar{p}^0$, dimensionless pressure (referred to $0.5\,\rho U_0^2$); ρ, liquid density; C_x, drag coefficient.

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